

KALMAN FILTER AND ITS APPLICATIONS

by

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Abstract

The origin and the various uses of Kalman Filters are explored in this paper. It is shown that Kalman Filters arise as a natural solution to the Wiener problem and can also be derived using Bayes' Rule but with stochastic assumptions on the auxiliary measurement. Econometric modeling, ARMA modeling and Regression modelling with stochastic coefficients are classical applications of Kalman Filters. It is also shown that combining forecasts can be an additional area of application of Kalman Filters. The utility of Kalman updating is shown to improve the combined forecasts of the Bersales-Mariano CPI model.

Keywords and phrases: dynamic system, prediction form, filtering form, statespace representation, auxiliary variable, mean square prediction error, primary forecasts, combined forecast, serial correlation.

1. INTRODUCTION

Kalman (1960) viewed a random function of time as an output of a dynamic system excited by gaussian noise. The dynamic system is described in terms of a state which contains the necessary information about the past behavior of the system to predict the future behavior. The system considered takes the form

$$\begin{aligned} \mathbf{X}_{t+1} &= \boldsymbol{\mu}_{t+1} + \boldsymbol{\Phi}_{t+1,t} \mathbf{X}_t + \mathbf{U}_{t+1} \\ \mathbf{Y}_t &= \mathbf{n}_t + \mathbf{M}_t \mathbf{X}_t + \mathbf{V}_t \end{aligned}$$

known as the prediction form and

$$\begin{aligned} \mathbf{X}_{t+1} &= \boldsymbol{\mu}_{t+1} + \boldsymbol{\Phi}_{t+1,t} \mathbf{X}_t + \mathbf{U}_{t+1} \\ \mathbf{Y}_{t+1} &= \mathbf{n}_{t+1} + \mathbf{M}_{t+1} \mathbf{X}_{t+1} + \mathbf{V}_{t+1} \end{aligned}$$

which is known as the filtering form. The vector \mathbf{X}_t is $p \times 1$ and known as the state vector; $\boldsymbol{\Phi}_{t+1,t}$ is a $p \times p$ matrix, called the transition matrix; \mathbf{Y}_t is a $d \times 1$ vector ($d \leq p$), called the auxiliary variable; \mathbf{M}_t is a $d \times p$ matrix; \mathbf{U}_t and \mathbf{V}_t are the noise components and are of length p and d , respectively.

The classical example used in citing the use of Kalman Filters is in tracking satellites. \mathbf{X}_t can represent the position and speed of a satellite at time t with respect to a spherical coordinate system with origin at the center. Note that this can't be measured directly. \mathbf{Y}_t consists of measurements of distance from the satellites and accompanying angles from tracking stations around the earth. Information on \mathbf{X}_{t+1} can be provided for by \mathbf{Y}_{t+1} in the Kalman set-up. Today, Kalman Filters has found its way in many areas of application such as in engineering problems, quality control and prediction. We add another method --

combination of forecasts.

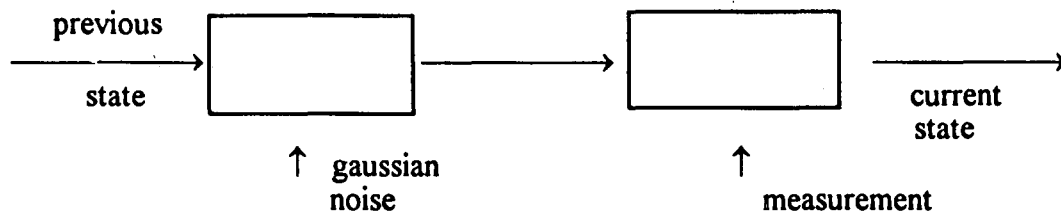


Fig. 1. Kalman Set-up.

The paper is arranged as follows: in Section 2 we will view Kalman Filters as a solution to the Wiener problem. In Section 3 we introduce how the Kalman set-up looks like in a Bayesian setting. We mention in sections 4 and 5 some areas of applications of Kalman Filters. Finally we look at an application of the Kalman theory in combining forecasts in Sections 6.

2. SOLUTION TO THE WIENER PROBLEM

Consider the vector differential system

$$\frac{d\mathbf{X}_t}{dt} = \mathbf{F}_t \mathbf{X}_t + \mathbf{D}_t \mathbf{U}_t$$

$$\mathbf{Y}_t = \mathbf{M}_t \mathbf{X}_t$$

The solution to this is the Wiener process given by

$$\mathbf{X}_{t+1} = \Phi_{t+1,t} \mathbf{X}_t + \mathbf{U}_t$$

$$\mathbf{Y}_t = \mathbf{M}_t \mathbf{X}_t$$

Kalman (1960) and later Kalman and Bucy (1961) showed that an estimate for \mathbf{X}_{t+1} is given by

$$\hat{\mathbf{X}}_{t+1|t} = \Phi_{t+1,t}^* \hat{\mathbf{X}}_{t|t-1} + \Delta_t^* \mathbf{Y}_t$$

whose estimation error is given by

$$\begin{aligned} e_{t+1|t} &= \mathbf{X}_{t+1} - \hat{\mathbf{X}}_{t+1|t} \\ &= \Phi_{t+1,t}^* e_{t|t-1} + \mathbf{U}_t \end{aligned}$$

where

$$\begin{aligned} \text{cov}(e_{t+1|t}) &= \mathbf{P}_t^* \\ \Delta_t^* &= \Phi_{t+1,t}^* \mathbf{P}_t^* \mathbf{M}_t' [\mathbf{M}_t \mathbf{P}_t^* \mathbf{M}_t']^{-1} \\ \Phi_{t+1,t}^* &= \Phi_{t+1,t} - \Delta_t^* \mathbf{M}_t \\ \mathbf{P}_{t-1}^* &= \Phi_{t+1,t}^* \mathbf{P}_t^* \Phi_{t+1,t}^* + \mathbf{Q}_{t+1} \\ \text{cov}(\mathbf{U}_t) &= \mathbf{Q}_t \end{aligned}$$

This estimate minimizes the Mean Square Prediction Error (MSPE).

For the Filtering problem the solution is given by

$$\hat{\mathbf{X}}_{t+1|t+1} = \Phi_{t+1,t}^* \hat{\mathbf{X}}_{t|t} + \Delta_{t+1}^* \mathbf{Y}_{t+1}$$

whose estimation error is given by

$$e_{t+1|t+1} = \Phi_{t+1,t} e_{t|t} + P_{t+1} U_{t+1}$$

where

$$\begin{aligned} \text{cov}(e_{t+1|t+1}) &= P_{t+1} \\ \Delta_{t+1} &= P_{t+1} M_{t+1}' (M_{t+1}' P_{t+1} M_{t+1})^{-1} \\ \Phi_{t+1,t} &= \Phi_{t+1,t} - \Delta_{t+1} M_{t+1} \Phi_{t+1,t} \\ P_{t+1} &= \Phi_{t+1,t}' P_t \Phi_{t+1,t} + R_{t+1} Q_{t+1}' R_{t+1}' \\ R_{t+1} &= I - \Delta_{t+1}' M_{t+1} \end{aligned}$$

3. KALMAN SOLUTION VIEWED IN THE BAYESIAN CONTEXT

Consider the same system but with stochastic assumptions on the auxiliary measurements, i.e.,

$$\begin{aligned} X_{t+1} &= \Phi_{t+1,t} X_t + U_{t+1} \\ Y_{t+1} &= M_{t+1} X_{t+1} + V_{t+1} \end{aligned}$$

and define $Y_{(t+1)}$ as the linear manifold generated by measurements Y_1, \dots, Y_{t+1} . Then

$$\begin{aligned} P(X_{t+1} | Y_{(t+1)}) &= \frac{P(Y_{(t+1)} | X_{t+1}) P(X_{t+1})}{P(Y_{(t+1)})} \\ &= \frac{P(Y_{(t+1)} | X_{t+1}, Y_{(t)}) P(X_{t+1} | Y_{(t)})}{P(Y_{(t+1)}, Y_{(t)})} \\ &= \frac{P(\varepsilon_{t+1} | X_{t+1}, Y_{(t)}) P(X_{t+1} | Y_{(t)})}{\int_{X_{t+1}} P(\varepsilon_{t+1}, X_{t+1} | Y_{(t)}) dX_{t+1}} \end{aligned}$$

where

$$\varepsilon_{t+1} = Y_{t+1} - Y_{(t+1)}$$

Meinhold and Singpurwalla (1983) considered this approach to find the solution obtained by Kalman by assuming that

$$\begin{aligned} X_{t+1} | Y_{(t)} &\sim N_p(\Phi_{t+1,t} \hat{X}_{t|t-1}, \Sigma_{t+1|t}) \\ V_t &\sim N_p(0, R_t) \\ U_t &\sim N_p(0, Q_t) \end{aligned}$$

where X_{t+1} , U_{t+1} and V_{t+1} are uncorrelated and

$$\Sigma_{t+1|t} = \Phi_{t+1,t} \Sigma_{t|t-1} \Phi_{t+1,t}' + Q_t$$

4. SOME AREAS OF APPLICATION

We present in this section some popular applications of the Kalman Filter in the modelling context.

Econometric Models

Consider the reduced form of a simultaneous system of equations given by

$$X_{t+1} = \Phi_{t,t+1} X_t + B_t Z_t + U_t$$

The interest lies in the estimation of X_{t+1} given observations (Z_t, X_t) , $t = t_0, \dots$. Suppose we obtained an external measurement Y_t from some survey results or computed from another econometric model, say of the form

$$Y_t = M_t X_t + C_t W_t + V_t$$

Then we can use the Kalman procedure to obtain estimates for X_{t+1} .

ARMA Models

Consider the multivariate ARMA model given by

$$\Phi(B)Y_t = \theta(B)e_t$$

where $\Phi(B)$ is the multivariate autoregressive polynomial operator and $\theta(B)$ the multivariate moving average operator. Akaike (1974) showed that the problem of identifiability pointed out by Quenouille (1957) can be avoided by analyzing the model using

$$\begin{bmatrix} Y_{t+1} \\ Y_{t+2|t+1} \\ \vdots \\ Y_{t+p|t+1} \end{bmatrix} = \begin{bmatrix} 0 & I_p & 0 & 0 \\ 0 & 0 & I_p & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \Phi_p & \Phi_{p-1} & \Phi_{p-2} & \Phi_1 \end{bmatrix} \begin{bmatrix} Y_t \\ Y_{t+1|t} \\ \vdots \\ Y_{t+p-1|t} \end{bmatrix} + \begin{bmatrix} I_d \\ \Phi_1 \\ \vdots \\ \Phi_{p-1} \end{bmatrix} e_{t+1}$$

or, in more compact form,

$$\begin{aligned} X_{t+1} &= \Phi_{t+1,t} X_t + \Delta e_{t+1} \\ Y_t &= (I_p, 0) X_t \end{aligned}$$

Thus, the Kalman filters set-up is a solution to the identifiability problem present in multivariate ARMA models

Regression Models

Consider the regression model with stochastic parameters given by

$$\begin{aligned} \beta_{t+1} &= \Phi_{t+1,t} \beta_t + U_t \\ Y_t &= X_t \beta_t + \varepsilon_t \end{aligned}$$

This type of set-up can be used to model structural changes that may occur in the regression parameters over time.

The optimal estimate of β_{t+1} , using the Kalman solution, is then given by

$$\beta_{t+1} = \Phi_{t+1}^* \hat{\beta}_{t+1} + \Delta_t^* Y_t$$

where

$$\begin{aligned} \Delta_t^* &= \Phi_{t+1}^* P_t^* X_t' [X_t P_t^* X_t' + R_t]^{-1} \\ \Phi_{t+1}^* &= \Phi_{t+1} - \Delta_t^* X_t \\ P_{t+1}^* &= \Phi_{t+1}^* P_t^* \Phi_{t+1}^* + Q_t + \Delta_t^* R_t \Delta_t^* \end{aligned}$$

5. KALMAN FILTERS IN COMBINING FORECAST

Consider the case when the primary forecasts $\hat{f}_{t+1}^{(1)}$ and $\hat{f}_{t+1}^{(2)}$ are generated by two different unknown models. It was shown empirically by Bersales, Mariano and Bonzo (1989) that a better forecast can be generated using

$$\hat{f}_{t+1} = \hat{\alpha}_{t+1} \hat{f}_{t+1}^{(1)} + (1 - \hat{\alpha}_{t+1}) \hat{f}_{t+1}^{(2)}$$

where $\hat{\alpha}$ is an Ordinary Least Squares (OLS) estimate of α in the regression equation given by

$$W_{t+1} = \alpha X_{t+1} + V_{t+1} \quad (5.1)$$

Here $W_{t+1} = Y_{t+1} - \hat{f}_{t+1}^{(1)}$ and $X_{t+1} = \hat{f}_{t+1}^{(1)} - \hat{f}_{t+1}^{(2)}$

When the combining weights are assumed to be time varying, the combining equation is given by

$$\hat{f}_{t+1} = \hat{\alpha} \hat{f}_{t+1}^{(1)} + (1 - \hat{\alpha}_{t+1}) \hat{f}_{t+1}^{(2)}$$

Under the assumptions that the forecast errors are uncorrelated and identically distributed, the solution is OLS and GLS otherwise.

Kalman filtering can be applied to this problem by considering the dynamic system given by

$$\begin{aligned} f_{t+1} &= \mu_{t+1} + \Phi_{t+1}^* f_t + u_{t+1} \\ Y_t &= n_t + M_t f_t + V_t \end{aligned}$$

Hence

$$\begin{aligned} \hat{f}_{t+1} &= \mu_{t+1} + \Phi_{t+1}^* \hat{f}_{t+1} + \Delta_t^* (Y_t - n_t) \\ \Delta_t^* &= \frac{\Phi_{t+1}^* P_t M_t}{M_t^2 P_t^*} \\ \Phi_{t+1}^* &= \Phi_{t+1} - \Delta_t^* M_t \\ P_{t+1}^* &= \Phi_{t+1}^* P_t^* \Phi_{t+1}^* + Q_t + \Delta_t^* R_t \Delta_t^* \end{aligned}$$

Consider the case when the combined forecasts are serially correlated. Suppose α_t follows AR(1), i.e.,

$$\alpha_{t+1} = \Phi_{t+1} \alpha_t + u_t, \quad |\Phi| < 1 \quad (5.2)$$

where u_t is white noise with variance σ_u^2 and u_t is uncorrelated with u_s and $\hat{f}_s^{(i)}$, $i = 1, 2, \dots$

Given $Y_t = \{Y_1, \dots, Y_t; f_1, \dots, f_t, i=1,2\}$ and assumptions

$$E(V_t) = 0 \forall t$$

$$E(V_t^2) = \sigma_t^2 \forall t$$

$$E(V_t V_s) = 0 \forall t \neq s$$

$$E(V_t u_s) = 0 \forall t \neq s$$

$\{\hat{f}_{t+1}^{(1)}, \hat{f}_{t+1}^{(2)}\}$ is independent of $\{\hat{f}_j^{(1)}, \hat{f}_j^{(2)}, j = 1, \dots, t\}$. Then the system given by (5.1), with α replaced by α_t , and (5.2) will yield

$$\hat{\alpha}_{t+1} = \phi_{t+1} \hat{\alpha}_{t+1} + \Delta_t^* W_t$$

where

$$\begin{aligned} \delta_t^* &= \frac{\phi_{t+1}^* P_t^* X_t}{X_t^2 P_t^* + \sigma_t^2} \\ \phi_{t+1}^* &= \phi_{t+1} - \phi_t^* X_t \\ P_{t+1}^* &= \phi_{t+1}^{*2} P_t^* + \sigma_w^2 + \delta_t^* \sigma_t^2 \end{aligned}$$

Hence, $W_{t+1} = \hat{\alpha}_{t+1} X_t$

or,

$$Y_{t+1} - \hat{f}_{t+1}^{(2)} = \hat{\alpha}_{t+1} (\hat{f}_{t+1}^{(1)} - \hat{f}_{t+1}^{(2)})$$

or,

$$Y_{t+1} = \hat{\alpha}_{t+1} \hat{f}_{t+1}^{(1)} + (1 - \hat{\alpha}_{t+1}) \hat{f}_{t+1}^{(2)}$$

and

$$E(Y_{t+1} - Y_{t+1})^2 = P_{t+1}^* [\hat{f}_{t+1}^{(1)2} + \hat{f}_{t+1}^{(2)2} - 2 \hat{f}_{t+1}^{(1)} \hat{f}_{t+1}^{(2)} + \sigma_{t+1}^2]$$

6. DATA ANALYSIS

Mariano (1987) used an estimated price equation which explained the dynamic behavior of monthly consumer price index (CPI) levels in terms of its own past values and cost-push and demand-pull factors. The following explanatory variables were used:

1. the value of CPI in the two immediately preceding months
2. the legislated minimum wage for the National Capital Region (WLNANC)
3. the average wholesale posted price of petroleum products as determined by government policy makers (PMOILD)
4. the interest rate on 91-day treasury bills (Day 91)
5. a black market premium for the U.S. dollar, measured in terms of the Hongkong peso-dollar banknote rate relative to the official exchange rate
6. a peso-denominated price import price index for non-fuels adjusted for tariffs
7. total domestic liquidity relative to real output, with lagged effects extending up to the three months

8. a peso-denominated over-all export price index
9. price ceilings in food items under price control

OLS using monthly data from January 1972 - March 1985 yielded a high coefficient of determination (99.96%). Furthermore, one-month-ahead forecasts for January 1983 - May 1985 yielded maximum error of 14.17 and minimum error of 0.20 and MSE of 48.75.

Bersales (1989) considered three quarterly models to generate price forecasts to illustrate how combining forecasts can improve poor primary forecasts. The price equations considered were as follows:

$$\text{(Quarterly Model 1)} \quad \text{CPI}_t = \beta_{10} + \beta_{11} \text{MERBM}_t + \beta_{12} \text{Import}_t + u_{1t}$$

$$\text{(Quarterly Model 2)} \quad \text{CPI}_t = \beta_{20} + \beta_{21} \text{WLNANC}_t + \beta_{22} \text{PMOILD}_t + \beta_{23} \text{Day 91}_t + u_{2t}$$

$$\begin{aligned} \text{(Quarterly Model 3)} \quad \text{CPI}_t = & \beta_{30} + \beta_{31} \text{MERBM}_t + \beta_{32} \text{IMPORT}_t + \beta_{33} \text{WLNANC}_t + \beta_{34} \text{PMOILD}_t \\ & + \beta_{35} \text{Day 91}_t + u_{3t} \end{aligned}$$

Here, the quarterly Hongkong peso-dollar banknote rate (MERBM) and quarterly data on imports (IMPORT) were used in lieu of the black market premium for the U.S. dollar and import price index. Note that the variables considered in the 3 price equations are just subsets of the original variables used by Mariano, hence poorer forecasts were anticipated. Also, Quarterly Models 1 and 2 are just subsets of Quarterly Model 3. It was shown that combining forecasts from Quarterly Models 1 and 2 results in a better forecast than that of Quarterly Model 3. However, the combined forecasts exhibited serial correlation.

Combination of the quarterly forecasts from Quarterly Model 3 and the aggregate of monthly forecasts generated by the model

$$\text{CPI}_{t,s} = \beta_0 + \beta_1 \text{CPI}_{t,s-1} + \beta_2 \text{CPI}_{t,s-2} + \beta_3 \text{PMI}_{t,s} + u_{t,s}$$

was done, where $\text{CPI}_{t,0} = \text{CPI}_{t-1,3}$

$$\text{CPI}_{t,-1} = \text{CPI}_{t-1,2}$$

The $u_{t,s}$ are independent and identically distributed with

$$\text{cov}(u_{3t}, u_{3s}) = \begin{cases} \sigma^2, & t = s \\ 0, & t \neq s \end{cases}$$

It was shown that the combined forecasts from these models has smaller MSE than that of the Quarterly Models 1 and 2.

The combined forecasts for these models were re-examined and Kalman updating was introduced. The two models considered were:

$$\begin{aligned} \text{(Model 1)} \quad f_{t+1} &= \mu_{t+1} + \Phi_{t+1,t} f_t + u_{t+1} \\ \text{CPI}_t &= n_t + M_t f_t + V_t \end{aligned}$$

$$\begin{aligned} \text{(Model 2)} \quad \alpha_{t+1} &= \emptyset \alpha_t + u_{t+1} \\ \text{CPI}_t &= \alpha_t \hat{f}_t^{(1)} + (1 - \alpha_t) \hat{f}_t^{(2)} + V_t \end{aligned}$$

For the combined forecasts of Quarterly Models 1 and 2, Kalman updating produced improved combined forecasts by using Models 1 and 2, but Model 1 failed to destroy the serial correlation present in the forecasts. The graph of the forecasts of Model 2 shows that the forecasts are criss-crossing the graph of the true CPI values: an indication that the problem of serial correlation was solved.

Kalman updating using model 2 also produced improved forecasts for the combined forecasts from Quarterly Model 3 and aggregate monthly model. However, Kalman updating using Model 1 was not that successful as was shown by the MSE. It is worthwhile to note that the use of Model 2 has dampened the presence of the serial correlations existing between the original combined forecasts. For a listing of the data used see Bersales (1990).

REFERENCES

- Akaike, H.(1974), Markovian representation of stochastic processes and its application to the analysis of autoregressive moving average processes, **Annals of the Institute of Statistical Mathematics**, 26, 363-387.
- Bersales, L.G.S.(1990), Optimal combination of forecasts, Dissertation University of the Philippines Diliman, Quezon City.
- Bersales, L.G.S., Mariano, R.S., and Bonzo, D.I.B.C. (1990), Combination of quarterly and monthly forecasts. **Proceedings of the Joint Conference in Statistics**.
- Box, G.E.P., and Jenkins, G.M.(1970), **Time Series Analysis and Control**. San Francisco: Holden Day.
- Diderrich, G.T.(1985), The Kalman Filter from the perspective of Goldberger-Theil estimators, **The American Statistician**, 39, 193-198.
- Downing, D.J., Pike, D.H., and Morrison, G.W.(1980), Application of the Kalman Filter to inventory control, **Technometrics**, 22, 17-22.
- Kalman, R.E.(1960), A new approach to linear filtering and prediction Problems, **Journal of Basic Engineering**, 82, 34-45.
- Kalman, R.E., and Bucy, R.S.(1961), New results in linear filtering and prediction theory, **Journal of Basic Engineering**, 83, 95-108.
- Mariano, R.S., and Schleicher, S.(1972), On the Use of Kalman Filters in Economic Forecasting, Discussion Paper 247.
- Meinhold, R.J., and Singpurwalla, N.D.(1983), Understanding the Kalman Filter, **The American Statistician**, 37, 123-127.
- Morrison, G.W., and Pike, D.H.(1977), Kalman Filtering applied to statistical forecasting, **Management Science**, 23, 768-774.

Sinsioco, C.S., Bautista, O.M., Bonzo, D.I.B.C., and Santos A.B., Multivariate Time Series Analysis of Selected Agricultural Variables" Proceedings of the Joint Conference in Statistics, 1990.

Swamy, P.A.V.B., and Tinsley, P.A.(1980), Linear prediction and estimation methods for regression models with stationary stochastic coefficients, **Journal of Econometrics**, 12, 103-142.

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Chang	Reynaldo	P	Duka	Marichu	F
Chua	Rosario	C	Dumapit	Pastora Gay	C
Cinto	Antonio	V	Duran	Heidi	F
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Collado	Roselle	V	Engracia	Luisa	T
Conti	Edna	B	Enriquez	Victoria Pauline	B
Cordial	Jenette	H	Erica	Carmelita	N
Costales	Ceferino	D	Esciamad	Katrina	
Cruz	Agustina	T	Escobillo	Evangeline	M
Cruz	Edgardo	D	Escuadro	Rico Hipolito	R
Cruz	Mananita	H	Esleta	Azucena	P
Cruz	Raul	C	Espeleta	Judith	T
Cruz	Remedios	A	Espinoza	Marites	
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Cubinar	Gloria	A	Esquivias	Minerva Eloisa	P
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